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Counting the Crowd

The Vatican said an historic 3.7 million people were at the Sunday event, an eye-popping number that would have made it the second largest papal Mass ever. . . . The problem was, the count released by Vatican and Brazilian officials was a guesstimate that statisticians say grossly inflated the crowd figures. The research director of Datafolha, one of Brazil’s top polling and statistic firms, said that based on the size of the crowd area and reasonable density estimates, he would put Sunday’s turnout at between 1.2 million and 1.5 million people. . . .

McPhail [an emeritus professor of sociology at the University of Illinois who has studied crowd counts for four decades] first chuckled when he heard the Vatican’s crowd estimate in an area of Copacabana beach and adjoining streets that encompassed about 497,000 square meters (594,400 square yards).

By the Vatican’s count, the crowd density throughout the entire area would have been 7.4 people per square meter, which wouldn’t allow for movement of any kind, let alone the jumping, arm waving, singing and dancing seen at the papal events. . . .

Source: “Numbers Don’t Add Up for Papal Mass Crowd Count,” Associated Press, Aug. 3, 2013, <http://newsinfo.inquirer.net/457893/numbers-dont-add-up-for-papal-mass-crowd-count>

1. The article states that 3.7 million people may have attended the papal Mass in an area of 497,000 square meters. By this count, what was the crowd density? Does this number match that stated in the clip?
2. Assume that the crowd area was a rectangular region with the beach along one side. The full article states that the beach is 4 km long. How wide was the crowd area?
3. The firm Datafolha and other researchers say that a good rule is 2 to 3 people per m^2 at a packed event. Given the size of the crowd area, find an interval to estimate the number of people at the event.
4. The research director for Datafolha estimated that there were between 1.2 million and 1.5 million people at the event. According to these estimates, what would be a reasonable range of the density of the crowd?
5. An example from the Programme for

International Student Assessment (PISA) reads:

At a rock concert, a rectangular field $100m \times 50m$ was reserved for fans to stand. The concert was sold out. Approximately how many fans were in attendance?

- A. 2000
- B. 5000
- C. 20,000
- D. 50,000
- E. 100,000

Given your answers to the previous questions, determine which of these answers is the most reasonable.

6. If the general rule is 2 to 3 people per m^2 in a packed event, what is a better estimate of the number of people at the concert in question 5?
7. Determine a good rule for determining the density of people at a packed event.

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Taxing the Garden

. . . Since 1982, the Garden [Madison Square]—an enormously successful, for-profit business—has not paid a dime to New York City in property tax for the land and the building it occupies. . . . A state law enacted 32 years ago granted the Garden a tax exemption that has no expiration date, a favor given to no other property in the state.

. . . For the tax year that begins in July [2014], the Garden will save “roughly \$54 million” in property taxes, according to George Sweeting, the deputy director of the city’s Independent Budget Office. [The previous] year, the savings were \$17 million. In 2002, it saved \$6.9 million. The deal is now worth nearly eight times more because the value of land in Manhattan has soared and because the Garden has invested heavily in renovating the building.

Cumulatively, the Garden has saved \$350 million since 1982.

. . . Even with a 40-year streak of futility, the Knicks are regarded as the most valuable franchise in basketball, worth \$1.4 billion; the Rangers are ranked the second most valuable hockey team, at \$850 million. . . .



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Source: Jim Dwyer, “32-Year Streak at the Garden: No Taxes Paid,” *New York Times*, May 20, 2014, <http://www.nytimes.com/2014/05/21/nyregion/32-year-streak-at-the-garden-no-taxes-paid.html>

1. According to the *New York Times*, Madison Square Garden has saved \$350 million over 32 years.
 - (a) What were the average savings in property taxes per year? (Assume that the number \$350 million has three significant figures.)
 - (b) The New York Knicks and the New York Rangers, who play in Madison Square Garden, are worth \$1.4 billion and \$850 million, respectively, according to the clip. Generally speaking, how might this tax agreement with Madison Square Garden affect the value of these teams?
2. Manhattan blocks are in the shape of a rectangle whose shorter side is approximately $\frac{1}{16}$ mile. Use an aerial view of Madison Square Garden to find the area, in square feet, of the square plot on which the Garden lies.
 - (a) What was the assessed property
3. The cylinder that is Madison Square Garden has a diameter of 400 feet.
 - (a) What is the volume of the cylinder if its height is half its radius?
 - (b) A human being of average size occupies about 0.07 cubic meters in volume. Approximately how many humans can fill Madison Square Garden?
 - (c) The venue advertises a seating capacity of approximately 20,000 people. Why is this value so different from that computed in part 3(b)?
4. The clip lists the Garden’s property tax savings as \$17 million for the 2012–13 tax year. The relevant property tax rate at the time was 10.288%. Use this information to answer the following questions:
 - (a) What was the assessed property
5. Madison Square Garden is not the only property exempt from property taxes—so are schools! Research the “footprint” of your school and find the commercial tax rate in your area. What would the annual taxes be for your school if it were subject to property taxes?
 - (a) By what percentage did the assessed value of Madison Square Garden increase as a result of the renovation?

Counting the Crowd answers

1. The crowd density is approximately 7.4 people per m², which matches the figure in the article:

3,700,000 people/497,000 m² ≈ 7.4 people/m²

2. The crowd area would be approximately 124 m wide:

lw = A
4000 m (w) = 497,000 m²
w = 124.25 m

3. There would be between 994,000 and 1,491,000 people at the event:

(2 people/m²)(497,000 m²) = 994,000 people
(3 people/m²)(497,000 m²) = 1,491,000 people

4. The density of the crowd would range between 2.4 people/m² to 3.0 people/m²:

1.2 million people/497,000 m² ≈ 2.4 people/m²
1.5 million people/497,000 m² ≈ 3.0 people/m²

5. When we determine the crowd density, an estimate of 20,000 people seems to be the most reasonable, allowing for 4 people/m². This is a little more than the suggested ratio in previous questions, which makes sense if the event were packed by being sold out.

The area of the field is (100m)(50m) = 5000 m². The density for each potential answer follows:

- A. 2000 people/5000 m² = 0.4 people/m²
- B. 5000 people/5000 m² = 1 person/m²
- C. 20,000 people/5000 m² = 4 people/m²
- D. 50,000 people/5000 m² = 10 people/m²
- E. 100,000 people/5000 m² = 20 people/m²

6. With 2 to 3 people per m², there would be 10,000 to 15,000 people at

the rock concert:

(2 people/m²)(5000 m²) = 10,000 people
(3 people/m²)(5000 m²) = 15,000 people

7. Figures 1–5 (“Counting the Crowd”) show examples of students standing within an area of 4 m². Figure 1 shows a density of 1 person per m²; figure 2 shows 2 people per m²; figure 3 shows 3 people per m²; figure 4 shows 4 people per m²; and figure 5 shows 7 people per m².

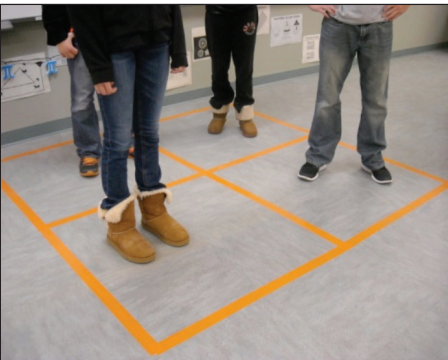


Fig. 1 (“Counting the Crowd”)

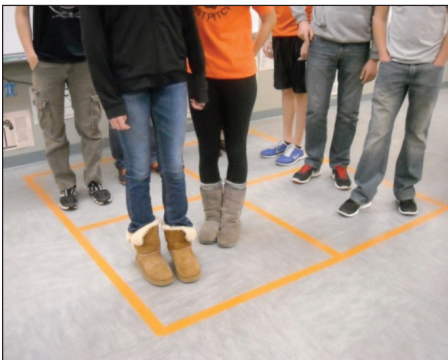


Fig. 2 (“Counting the Crowd”)

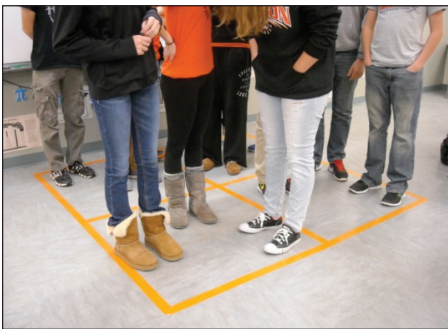


Fig. 3 (“Counting the Crowd”)



Fig. 4 (“Counting the Crowd”)



Fig. 5 (“Counting the Crowd”)

Students evaluating crowd densities report that 1 to 2 people per m² is not very crowded and that 3 to 4 people per m² is reasonable for a packed event. They report that 7 people per m² is impossible for any event. Have your students fill 4 m² so that they can see for themselves!

Taxing the Garden answers

1. (a) The average savings in property taxes per year is \$10,900,000:

(\$350,000,000 / 32) = \$10,937,500 ≈ \$10,900,000

The article provides only three significant figures, so we round to the nearest hundred thousand.

(b) The tax agreement with Madison Square Garden decreases each team’s expenditures and so most likely increases their value. However, a team’s value also depends on many other factors,

such as star players, performance, and ticket holders’ interest. As a result, it is unclear exactly how much of each team’s value is determined by the tax exemption.

2. Using Google Maps™, we see that the circular Garden seems to fit on a square roughly two “short side” blocks long (see fig. 1 [“Taxing the Garden”]). Each short side is equal to 1/16 mile. Two short sides total 1/8 mile, or 5280/8 = 660 ft. The area of the square would be 660² = 435,600 ft.².

3. (a) The volume of the cylinder is approximately 12,600,000 ft.³:

V_cylinder = π • r² • h
= π • (400 / 2)² • 100
= 4,000,000 • π
≈ 12,566,371 ft.³

(b) Note that 1 meter comprises approximately 3.28 ft. Thus, 35.29 ft.³ is approximately equivalent to 1 m³. Hence, 0.07 m³ is approximately 2.50 ft.³. On the basis of this estimate, approximately 12,600,000/2.50—close to 5 million people!—can fill Madison Square Garden.

(c) The calculation in part 3(b) is far from accurate. It assumes that there will be no space between people and that people can be stacked vertically (i.e., on top of one another).

4. (a) The assessed property value of Madison Square Garden for the 2013–14 tax year was approximately \$170,000,000 (using two significant digits):

0.10288x = \$17,000,000
x = \$17,000,000 / 0.10288
x = \$165,241,057.54

(b) Madison Square Garden would be worth approximately \$520 million for the 2014–15 tax year:

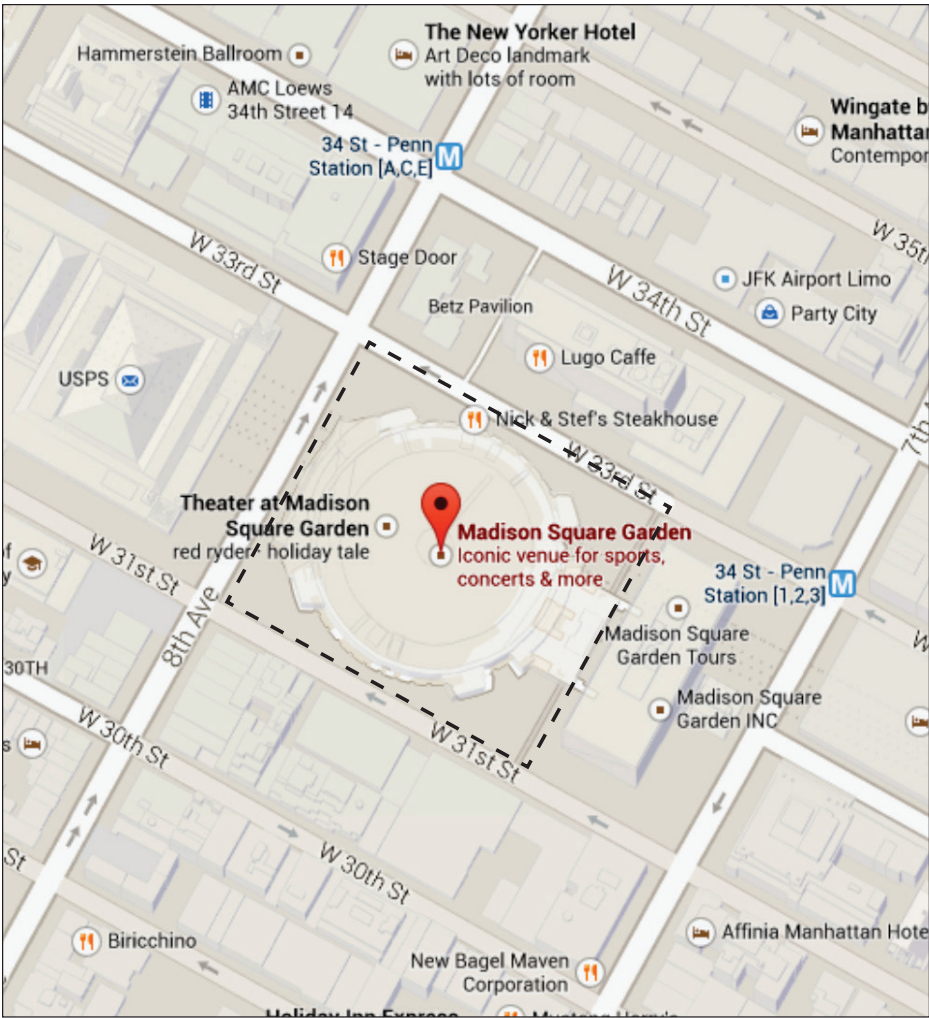


Fig. 1 (“Taxing the Garden”)

0.10323x = \$54,000,000
x = \$54,000,000 / 0.10323
x = \$523,103,748.91

(c) As a result of the renovations, the value of the Garden jumped from approximately \$170 million to \$520 million. The percent increase is approximately 200%:

(520 – 170) / 170 • 100 = 205.88%

5. Answers will vary. County tax departments (and possibly the school district offices) will have assessed values for the school property and commercial tax information.



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High School in Oregon, Wisconsin. He is interested in having students discover the mathematics in the world around them. JULIA MORRIS, EMILY BROUDE, and ROBERT LESSER, who submitted “Taxing the Garden,” are graduates of Hastings High School in Hastings on Hudson, New York. They can be reached through their former mathematics teacher, Greg Stephens, at stephensg@hohschools.org. They are interested in exploring how math can be applied to the real world as well as learning more abstract concepts. They are looking forward to furthering their mathematical education at the collegiate level.